REAL OPTION VALUE

CHAPTER 10 REAL DEBT GAMES

An appropriate extension of real options and game theory is to the context of lender-borrowers, where is no investment pre-emption, but contrasting interests and conflict. Various authors including Leland (1994) examine the effect on equity, debt and asset values, (and on the investment decisions), of differences in capital structures, given taxes, bankruptcy costs and possible negotiations among borrowers and lenders. In funding arrangements, typically a lender does not share in the upside potential of a borrower, so burdening the assets of an enterprise with debt turns the equity of the borrower into a call option, with the debt as the equivalent exercise price. First, equity is viewed as a simple perpetual call option, with debt as a perpetuity. The borrower has the proprietary right to default. Then, we examine who wins and who loses as different management types implement certain actions subsequent to the initial funding. Finally, this structure is extended to cover possible "fair" renegotiated debt arrangements.

10.1 REAL DEBT OPTIONS

Suppose firm asset values follow a geometric Brownian motion process, with constant volatility:

$$dV = \mu V dt + \sigma V dz \tag{10.1}$$

where μ is the growth rate or drift parameter; σ the volatility and dz the increment of a Wiener process. In a risk neutral world, or in perfect hedging which earns the riskless return, $\mu = r - \delta$, where δ is the asset yield. Suppose there is a perpetual claim, D, on the firm that continuously pays a coupon C, when the firm is not bankrupt, the value of that claim can be represented by the following differential equation:

$$\frac{1}{2}\sigma^2 V^2 \frac{\partial^2 D}{\partial V^2} + (r-\delta)V\frac{\partial D}{\partial V} - rD + C = 0 \tag{10.2}$$

Equation (10.2) is a second order linear ordinary differential equation ("ODE") with the solution:

$$D(V) = A_0 + A_1 V^{\beta_1} + A_2 V^{\beta_2} \tag{10.3}$$

where β_2 is the negative root solution of the characteristic quadratic equation[1]:

$$\frac{1}{2}\sigma^2 \beta(\beta - 1) + (r - \delta)\beta - r = 0 \tag{10.4}$$

and therefore β_2 is:

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0 \tag{10.5}$$

The boundary conditions for the debt claim are shown in Appendix 10A. Because $V^{\beta 2}=0$ as V approaches infinity, $A_0=C/r$. If $V\rightarrow\infty$, debtholders receive at best D=C/r (implying that $A_1=0$, because debtholders receive none of the upside). If there is bankruptcy, debtholders receive V less the bankruptcy costs α (expressed as a proportion of V), $(1-\alpha)V_B$, where $V_B=V$ at the point of bankruptcy. Where V_B is not pre-specified, it is the solution of the first derivative of E* (the equity value =V*-D*) with respect to V equal to 0. In this case, the bankruptcy trigger is chosen by the managers (or shareholders) not the debtholders. As derived in Appendices 10A and 10B, there is a closed-form solution for D* (real debt value), V*(real asset value) and E* (real equity value).

$$D^* = \frac{C}{r} + \left[(1-\alpha)V_B - \frac{C}{r}\right](\frac{V_B}{V})^{-\beta_2} \tag{10.6}$$

$$V^* = V + \frac{\tau C}{r}\left[1 - \left(\frac{V_B}{V}\right)^{-\beta_2}\right] - \alpha V_B \left(\frac{V_B}{V}\right)^{-\beta_2} \tag{10.7}$$

$$E^* = V - (1-\tau)\frac{C}{r} + \left[(1-\tau)\frac{C}{r} - V_B\right]\left(\frac{V_B}{V}\right)^{-\beta_2} \tag{10.8}$$

$$V_B = \left[(1-\tau)\frac{C}{r}\right]\left(\frac{-\beta_2}{1-\beta_2}\right) \tag{10.9}$$

The spreadsheet "proof" that equation 10.6 is the solution of equation 10.2 is shown in Figure 10.1a.

[1] See Appendix 4A for the basic solution of a quadratic equation.

Figure 10.1a

	A	B	C	D
1		ODE PROOF for DEBT CLAIM		
2				
3	INPUT			
4	V	40		
5	C	4		
6	D	66.67		
7	δ	0.00		
8	μ=r–δ	0.06		
9	σ	0.20		
10	α	0.50		
11	τ	0.35		
12	r(f)	0.06		
13	OUTPUT			
14	V_B	32.50	(1-B11)*(B5/B12)*(-B16/(-B16+1))	10.9
15	D* (V)	39.62	(B5/B12)+((1-B10)*B14-(B5/B12))*(B14/B4)^-B16	10.6
16	β_2	-3.00	(0.5-(B12-B7)/(B9^2)-SQRT((0.5-(B12-B7)/(B9^2))^2+2*B12/(B9^2)))	10.5
17				
18	ODE	0.0000	0.5*(B9^2)*(B4^2)*B20+B12*B4*B19-B12*B15+B5	10.2
19	D'(V)	2.0282	B16*((1-B10)*B14-(B5/B12))*(1/B4)*(B14/B4)^-B16	A6
20	D''(V)	-0.2028	((B16^2)-B16)*((1-B10)*B14-(B5/B12))*(1/(B4^2))*(B14/B4)^-B16	A7

D*(V) is from equation 10.6, D'(V) and D''(V) are from Appendix 10A. The calculation for D*(V) and the first and second derivatives are substituted into equation 10.2, cell B18=ODE, which is equal to zero.

The conflicts between borrowers and lenders might be viewed in terms of the sensitivities of the various elements of the real balance sheet to changes in the nominal asset value, asset volatility, taxation and interest rate (possibly under the control of managers and/or government). Sadly, bankruptcy costs in most countries are the province of lawyers and accountants. In the base case, V=40, nominal debt=66.7 (coupon of 4 divided by riskless rate of 6%), asset yield is nil, asset volatility is 20%, bankruptcy cost =50%V, and tax rate is 35%. Since the straight debt value exceeds V even considering the tax benefit of debt, real equity value is small (2.48), but reflecting the call aspect of equity (or the equivalent put option written by debtholders) and the value of tax benefits over bankruptcy costs.

A comparison of nominal asset value, nominal debt value, and nominal equity value to real asset, real debt and real equity value is shown in Table 1. The B column is the real option value, column C the nominal account, column D the tax benefit and option value, column E the value of the bankruptcy costs, and column F the SOTP (sum of the parts, columns C+D+E). In the base case, nominal asset value of 40 is enhanced (+10.82) (D30) by tax relief on interest payments on the embedded debt, but lower due to the current value of bankruptcy assets of 8.72 (E30). Real debt value is 27.04 lower than the nominal debt value due to current put value (D31). Nominal equity is negative (V-ND=40-66.6=-26.7), but E* is enhanced due to the tax relief on interest payments and the call option aspect (D32) and avoiding the value of the bankruptcy cost (E32).

	A	B	C	D	E	F
28	**Table 1: Decomposition of Real Values**					
29			NOMINAL	TB+ROV	BC	SOTP
30	V*	42.10	40.00	10.82	-8.72	42.10
31	D*	39.62	66.67	-27.04		39.62
32	E*	2.48	-26.67	23.33	5.81	2.48

Figure 10.1b shows that as V increases, especially past the optimal endogenous V bankruptcy trigger (V_B), the real value of the debt increases (as the put option written by the debtholders decreases eventually to a very small amount). As V increases, the real value of assets increases primarily because of the value of tax relief on interest payments, and lower probability of bankruptcy costs. As V increases, equity also increases, so that at high V, equity more or less equals the difference between V* and D*=Nominal Debt. So it is in the best interest of both borrowers and lenders that nominal asset value increases (perhaps best thought of as underlying property value, which then is enhanced in a structure where tax relief is given on interest payments).

Figure 10.1b

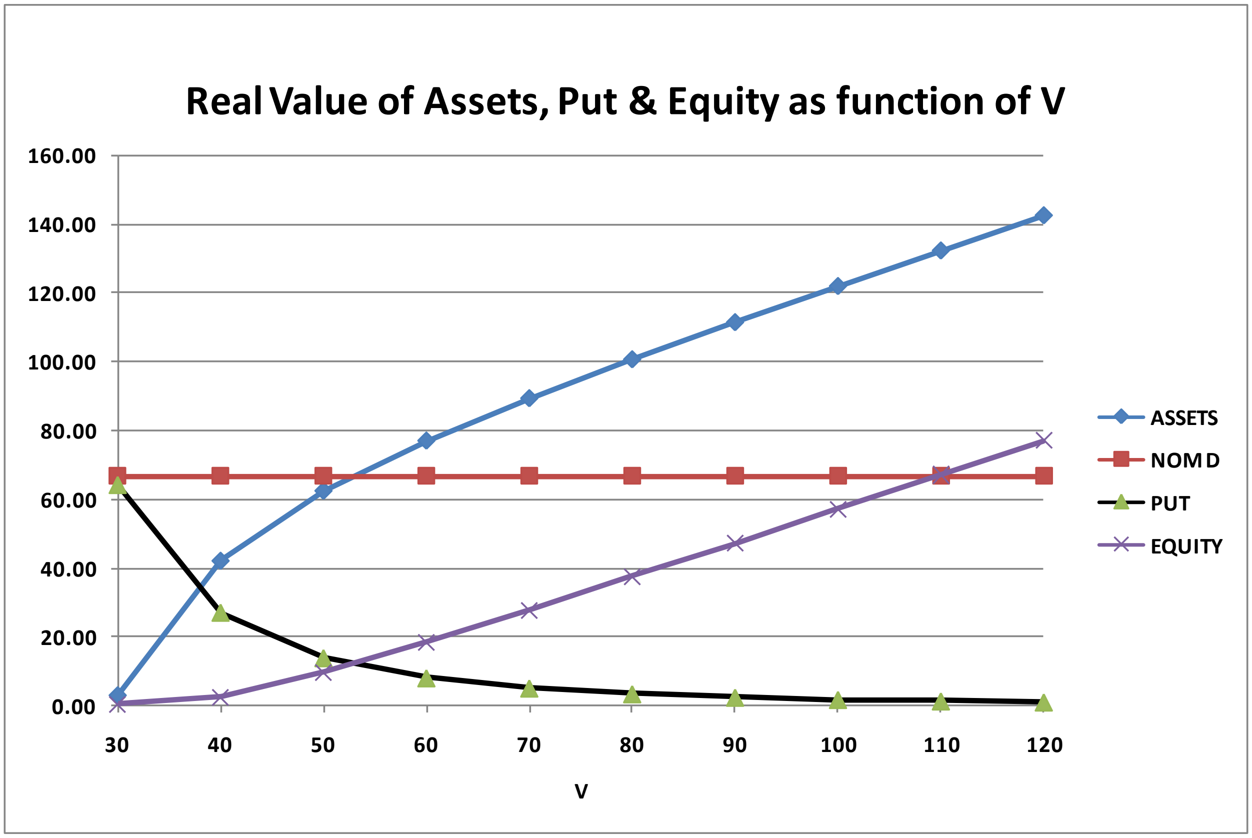
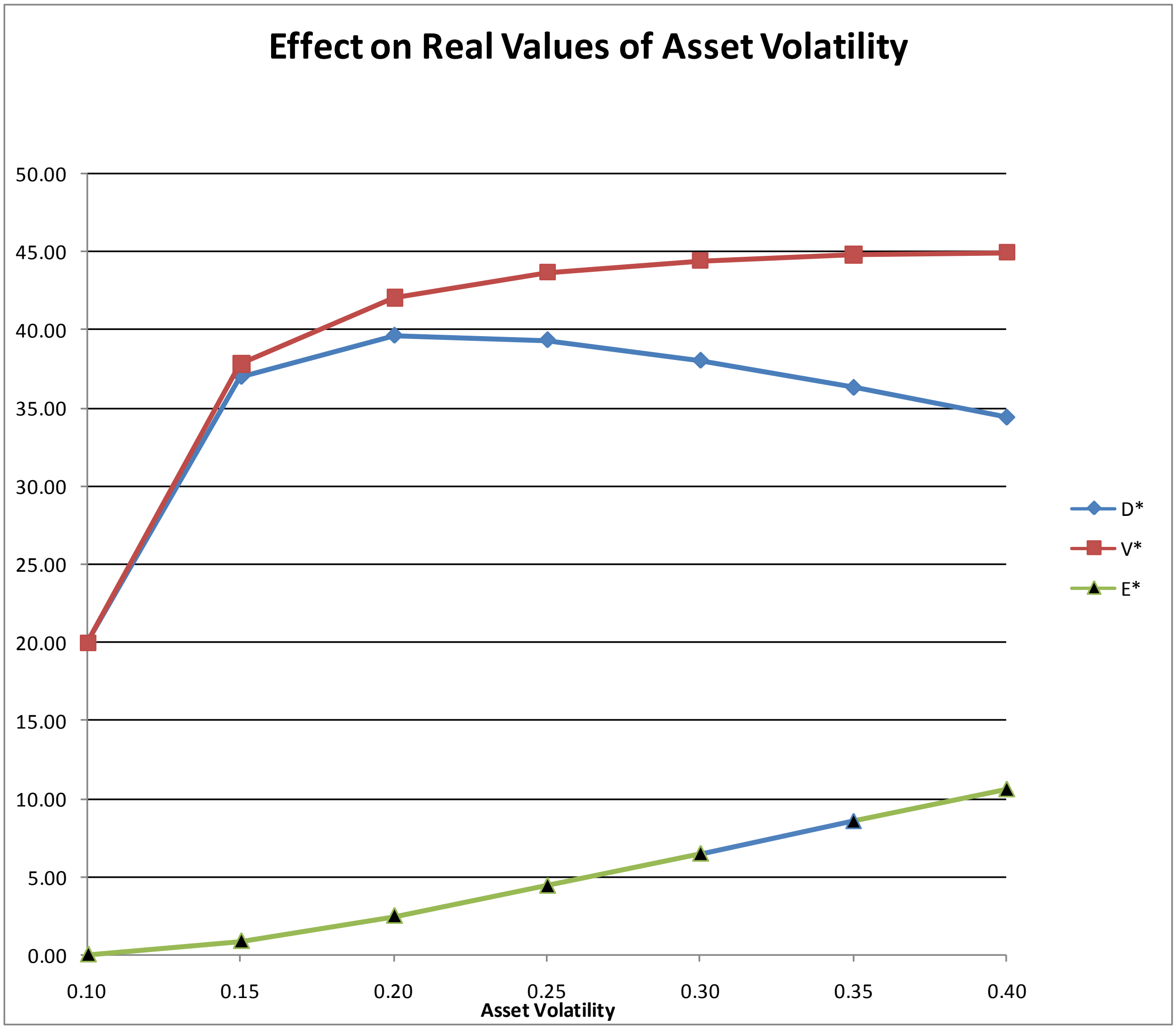


Figure 10.1c shows the sensitivity of real asset, debt and equity value as volatility increases (vega). With the base parameters, at a volatility of 10%, $V=V_B$, implying that bankruptcy occurs, so real debt value is very low, and equity value is nil. As volatility increases from 15% to 40%, V_B increases so the tax benefit from debt increases, and the value of bankruptcy costs decreases, so the asset value V* increases at a high rate from low volatility and then at a slow rate for high volatility. As volatility increases over 20%, D* decreases as V increasingly exceeds V_B, so management has less incentive to default. Volatility increases equity value E*, which increases from nearly nil at low volatility (since this is an out-of-the-money equity call option) to a high value at high volatility. An obvious conflict between shareholders and debtholders is regarding asset volatility.

Figure 10.1c



The impact of changes in the tax rate on real asset and real equity values is curious, and is shown in Figure 10.1d. When the tax rate decreases below 20%, $V^* < V_B$, so the managers should exercise the option to default. Then the value of the debt is 50%V due to the proportional bankruptcy costs. When the tax rate is increased over @20%, given the assumed parameters, real equity value increases slightly, due to the tax benefits of debt.

Figure 10.1d

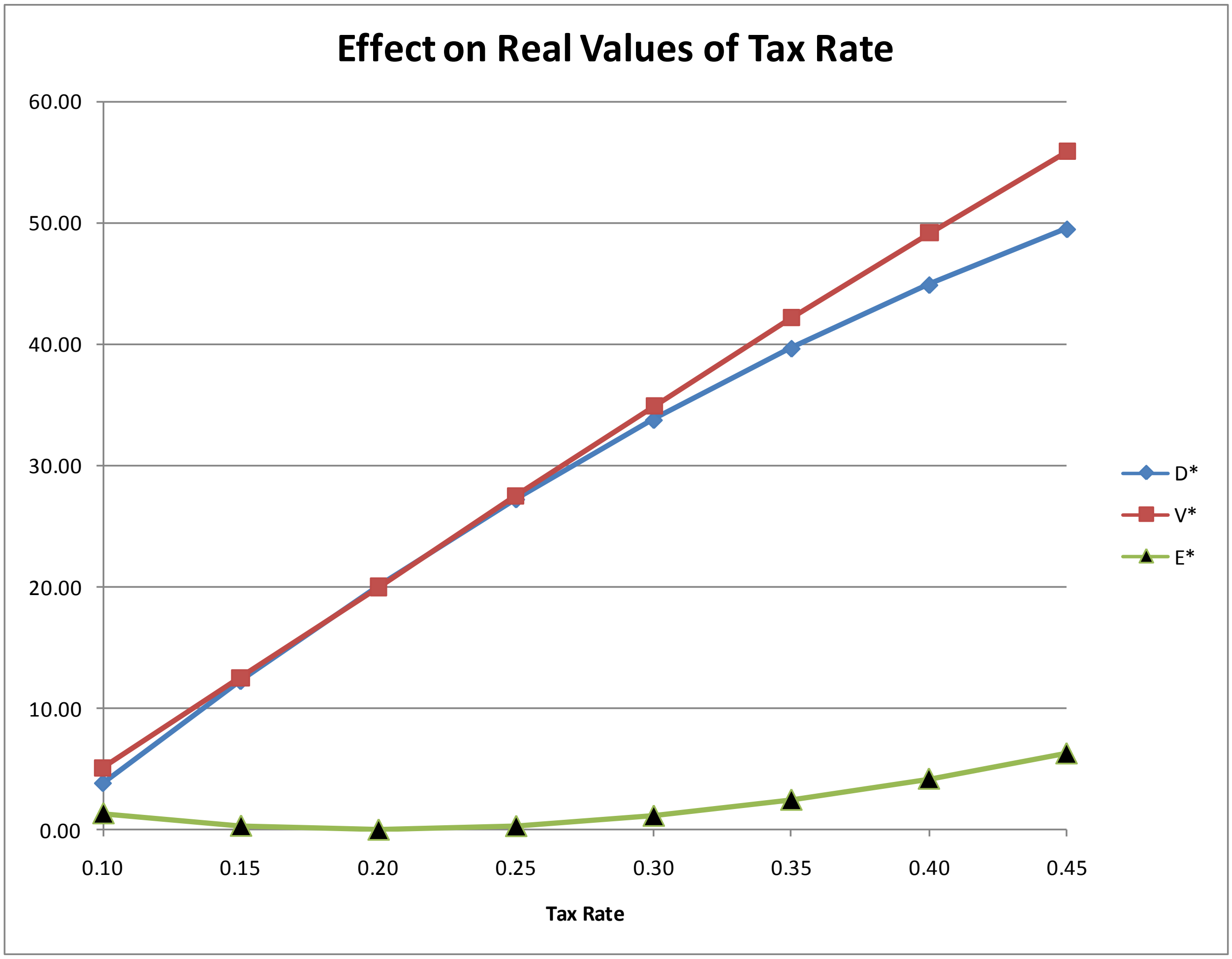
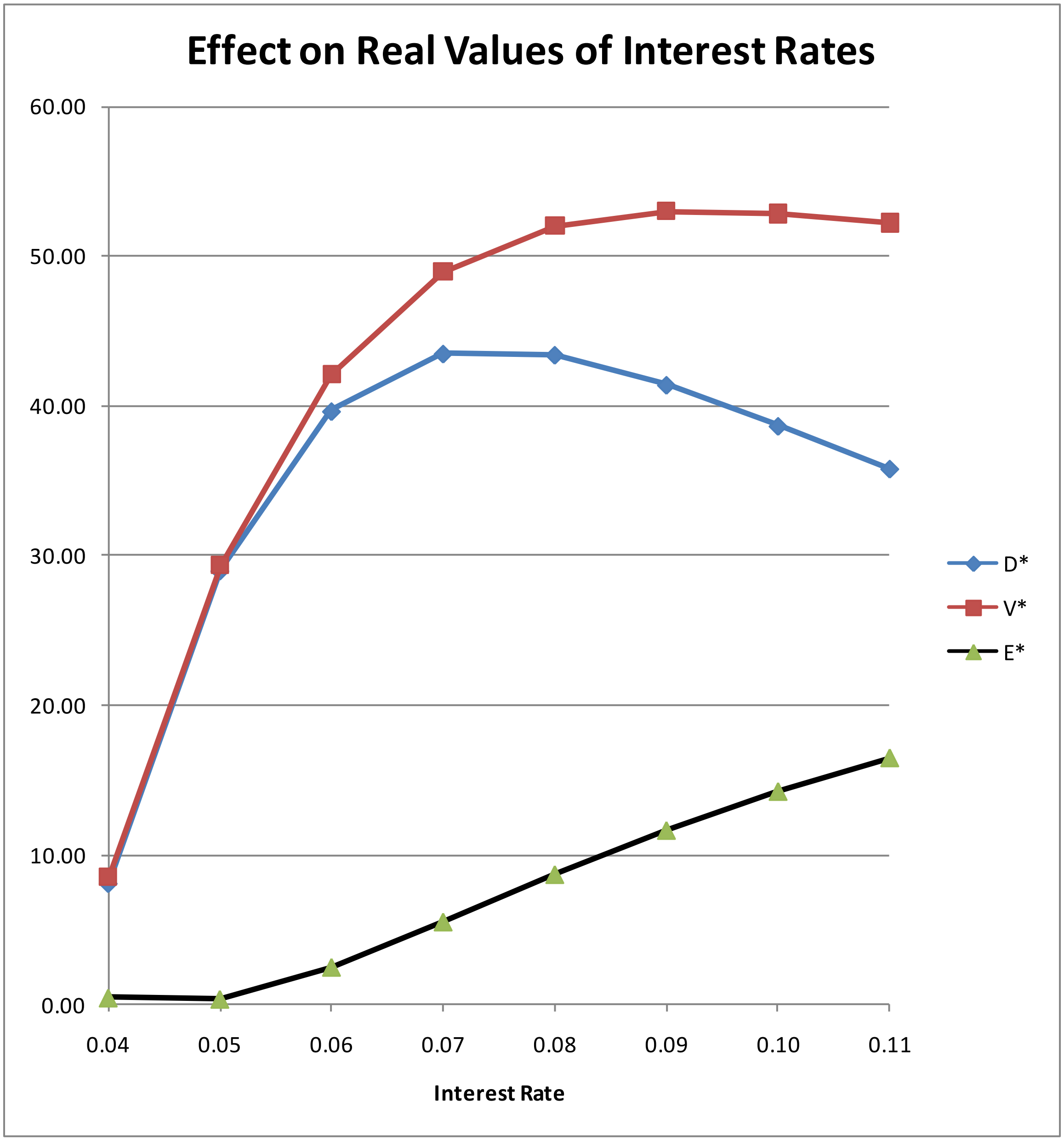


Figure 10.1e shows the effect of changes in the riskless interest rate on D*, V* and E*. Given these base parameter values, r<4% yields a V_B>V, implying an immediate optimal default. At higher interest rates, V_B declines, so all real values increase over a range. Given the fixed coupon on the embedded debt, there is a trade-off between the then nominal value of the debt (C/r) which falls, and the value of bankruptcy costs, which falls even more, so D* increases over a range. This assumes that V is not affected by changes in r.

Figure 10.1e



Assuming managers are primarily interested in raising real equity values from any actions after the firm issues the debt with a fixed coupon, actions that increase V, increase volatility, increase taxation beyond a 20% rate, or increase riskless interest rates beyond 4% are likely to be beneficial to shareholders (but not always to debtholders).

10.2 REAL EQUITY INCENTIVES

Shakey Enterprises Corporation ("SEC") has $67 nominal perpetual $4 coupon debt, and assets currently worth $40. In an immediate liquidation of SEC, the debt would be worth only $40 (less bankruptcy costs) and the equity holders would receive nothing. However, the debt is a perpetuity, so equity is really a perpetual call option on the asset value. Equity holders have the option of declaring bankruptcy at any time and also have unlimited time to pay off the debt anytime (if it is callable). Debtholders have in effect sold a put option to the equity holders. Since the nominal debt value exceeds the current asset value, this is an in-the-money American perpetual put option.

Figure 10.1f

	A	B	C	D	E	F
1		**Real Debt Strategy**				
2		Figure 10.1				
3		**BASE CASE**				
4		**ACCOUNTS**	**REAL**			
5	ASSETS	40.00	42.10		10.7	
6	DEBT	66.67	39.62		10.6	
7	EQ	-26.67	2.48		10.8	
8						
9	INPUT	BASE				
10	NOM V	40.00	40.00			
11	Coupon	4.00	4.00			
12	NOM Debt	66.67	66.67			
13	δ		0.00			
14	σ		0.20			
15	α		0.50			
16	τ		0.35			
17	r(f)		0.06			
18	OUTPUT					
19	V_B		32.50		10.9	
20	β_2		-3.00		10.5	
21						
22	V*	C10+(C16*C11/C17)*(1-(C19/C10)^-C20)-C15*C19*(C19/C10)^-C20				
23	D*	(C11/C17)+((1-C15)*C19-(C11/C17))*(C19/C10)^-C20				
24	E*	C10-(1-C16)*(C11/C17)+((1-C16)*(C11/C17)-C19)*((C19/C10)^-C20)				
25	V_B	(1-C16)*(C11/C17)*(-C20/(-C20+1))				
26	β_2	(0.5-(C17-C13)/(C14^2)-SQRT((0.5-(C17-C13)/(C14^2))^2+2*C17/(C14^2)))				

As shown in Figure 10.1f, the base case "accounting" assets V=40, nominal debt=66.67, so the nominal "accounting" equity is negative. But with an expected asset volatility of 20%, riskless interest rate of 6%, bankruptcy costs =50%V, a tax rate of 35%, the equity viewed as a call option is worth $2.48, the equivalent put option (plus bankruptcy costs) is worth $27.05, the firm's asset value with the embedded debt is worth $42.10, and the value of the debt is $39.62. Note that these game outcomes are in some cases not the same for creditworthy firms (or low LTV borrowers).

Suppose Shakey Enterprises has three possible sets of managers, a devious management DM, a stupid management SM and a super smart management SSM. All of these managers have an equity stake more valuable than the present value of their future compensation arrangements. In every case, management starts from the position in Figure 10.1f.

CASH OUT GAME

Now suppose the first strategy (a) of DM is to convert some assets into cash and then pay a $5 dividend to equity holders, including themselves. Figure 10.2 shows the result, with the equity

Real Debt Strategy		
Figure 10.2		
BASE CASE		
	ACCOUNTS	**REAL**
ASSETS	35.00	26.64
DEBT	66.67	26.30
EQ	-31.67	0.34
INPUT	BASE	
NOM V	**35.00**	**35.00**
Coupon	4.00	4.00
NOM Debt	66.67	66.67
δ		0.00
σ		0.20
α		0.50
τ		0.35
r(f)		0.06
OUTPUT		
V_B		32.50
β_2		-3.00

stake being reduced to $.34 which with the dividend equals $5.34, with the bondholders' stake being reduced significantly. V is close to V_B, so bankruptcy is imminent, reducing both V*and D* due to bankruptcy costs and loss of tax benefits.

DASH FOR CERTAINTY

Now suppose the SM is initially in charge, turns V into low volatility assets (perhaps by hedging), thereby reducing the asset volatility to 10% strategy (b). The equity value disappears, but the bondholders lose, since the real asset value has declined as shown in Figure 10.3.

Real Debt Strategy		
Figure 10.3		
BASE CASE		
	ACCOUNTS	**REAL**
ASSETS	40.00	20.00
DEBT	66.67	20.00
EQ	-26.67	0.00
INPUT	BASE	
NOM V	40.00	40.00
Coupon	4.00	4.00
NOM Debt	66.67	66.67
δ		0.00
σ		**0.10**
α		0.50
τ		0.35
r(f)		0.06
OUTPUT		
V_B		40.00
β_2		-12.00

DEVIOUS MANAGERS IN INEFFICIENT MARKETS

Now suppose instead of (a), the DM implements strategy (c), with a contribution of $10 from rather thick bondholders (perhaps some government), and invests the funds in a project so total asset volatility is 30%, where the NPV=0, so V increases by $10. While debtholders are slightly better off (total nominal debt of 67 is now worth 45, due to lower likelihood of bankruptcy, net debt is 35), equity value increases by $11.46 (13.94-2.48), as shown in Figure 10.4.

Real Debt Strategy		
Figure 10.4		
BASE CASE		
	ACCOUNTS	**REAL**
ASSETS	50.00	59.34
DEBT	66.67	45.40
EQ	-16.67	13.94
INPUT	BASE	
NOM V	50.00	50.00
Coupon	4.00	4.00
NOM Debt	66.67	66.67
δ		0.00
σ		**0.30**
α		0.50
τ		0.35
r(f)		0.06
OUTPUT		
V_B		24.76
β_2		-1.33

Bondholders will not put up with that DM for long. A wiser DM requests a $10 from debtholders and invests in 20% volatility projects with a positive NPV=+5, strategy (d), Figure 10.5.

Real Debt Strategy		
Figure 10.5		
BASE CASE		
	ACCOUNTS	**REAL**
ASSETS	55.00	70.17
DEBT	66.67	56.26
EQ	-11.67	13.90
INPUT	BASE	
NOM V	**55.00**	**55.00**
Coupon	4.00	4.00
NOM Debt	66.67	66.67
δ		0.00
σ		0.20
α		0.50
τ		0.35
r(f)		0.06
OUTPUT		
V_B		32.50
β_2		-3.00

Now equity value has increased by \$11.42 (more than 5.5 times) while debtholder value has increased by \$6.64 (16.64 -10). Is this a fair division of wise investing?

Now, suppose SSM initially runs SEC, and implement strategy (e) before DM or SM do strategies (b) or (d). Facing knowledgeable new bondholders, SSM issue \$16.67 nominal new bonds on the same terms as before (so the total coupon is \$5), and invest in a NPV +5 project, except that the new funds are raised on more or less "fair terms" equal to the pre-funding bond value (market value=39.62/66.7=59% nominal value). So \$16 nominal bonds are issued for \$10 (just \$.50 more than the old fair value), assets increase by \$10 plus NPV+\$5, shown in Figure 10.6.

Real Debt Strategy		
Figure 10.6		
BASE CASE		
	ACCOUNTS	**REAL**
ASSETS	55.00	64.23
DEBT	83.33	57.94
EQ	-28.33	6.29
INPUT	BASE	
NOM V	**55.00**	**55.00**
Coupon	**5.00**	**5.00**
NOM Debt	83.33	83.33
δ		0.00
σ		0.20
α		0.50
τ		0.35
r(f)		0.06
OUTPUT		
V_B		40.63
β_2		-3.00

Real equity has increased by \$3.81, old bonds have increased by @ \$6.92, and new bonds are worth @\$1.60 more than the issue price. This appears to be a win-win strategy primarily because everyone is better off due to the "fair terms" and the high positive NPV project investment.

PLAYING POLITICS

Governments control taxation and (to some extent) riskless interest rates. When firms are in financial distress, SSM appeal to the government (but don't fly in private jets to Washington, instead drive in environmentally friendly cars, or take the train) to raise income (but not capital gains) taxes, strategy (f) as shown in Figure 10.7. A tax rate of 50% would increase the benefit of deducting interest, so that the real asset value increases, accompanied by an increase in the real debt value (due to lower bankruptcy likelihood) and a significant increase in the real equity value. If debt is held by non-tax paying institutions, debtholders would be better off even after-tax.

Real Debt Strategy		
Figure 10.7		
BASE CASE		
	ACCOUNTS	**REAL**
ASSETS	40.00	62.14
DEBT	66.67	53.44
EQ	-26.67	8.70
INPUT	BASE	
NOM V	40.00	40.00
Coupon	4.00	4.00
NOM Debt	66.67	66.67
δ		0.00
σ		0.20
α		0.50
τ		**0.50**
r(f)		0.06
OUTPUT		
V_B		25.00
β_2		-3.00

What if the SSM appeal to the government to actually raise interest rates, strategy (g)?
As shown in Figure 10.8, nominal bond value decreases (assuming nominal D=C/r) but real debt value increases, as do real asset and real equity values, due to the tax benefit of the embedded debt at the fixed coupon. Curiously this appears to benefit everyone, assuming that V does not decline with slower economic activity due to higher interest rates.

Real Debt Strategy		
Figure 10.8		
BASE CASE		
	ACCOUNTS	**REAL**
ASSETS	40.00	48.97
DEBT	57.14	43.47
EQ	-17.14	5.50
INPUT	BASE	
NOM V	40.00	40.00
Coupon	4.00	4.00
NOM Debt	57.14	57.14
δ		0.00
σ		0.20
α		0.50
τ		0.35
r(f)		**0.07**
OUTPUT		
V_B		28.89
β_2		-3.50

Both of these actions have curious results, which are the opposite of government strategy in the current global financial markets meltdown.

SUPER SMART MANAGEMENT IN INEFFICIENT MARKETS

Alternatively, suppose SSM (this time with a small "free" equity stake) observe that originally the equity for this distressed company is selling in the stock market for only $1, perhaps because outside stockholders do not recognize real option values. Instead of paying a dividend of $5 like the first actions of DM, in strategy (h) SSM offers to repurchase all (except for the SSM stake) equity for $2, 100% more than the then market price as shown in Figure 10.9.

Of course, this is really a type of real options management buy-out since now SSM own SEC, and the SSM stake is worth $1.44. The total bondholders' stake is worth $4.49 less. SSM have not paid anything significant for their small stake (and hold at least one share that has not been repurchased) but are aware of real option values. SSM could compensate bondholders for some of this loss, and still come out ahead, due to the real options myopia of former stockholders.

Real Debt Strategy		
Figure 10.9		
BASE CASE		
	ACCOUNTS	**REAL**
ASSETS	38.00	36.57
DEBT	66.67	35.13
EQ	-28.67	1.44
INPUT	BASE	
NOM V	38.00	38.00
Coupon	4.00	4.00
NOM Debt	66.67	66.67
δ		0.00
σ		0.20
α		0.50
τ		0.35
r(f)		0.06
OUTPUT		
V_B		32.50
β_2		-3.00

What other strategies can you think of for the SSM?

10.3 FAIR DEBT RENEGOTIATION

In all of these strategies debtholders suffer a loss, from 13.22 to 46.67 (immediate liquidation).

	TABLE II	
	VALUE	**OLD DEBT LOSS**
NOMINAL	66.67	
BASE	39.62	27.04
RS a	26.30	40.37
RS b	20.00	46.67
RS c	35.40	31.27
RS d	46.26	20.40
RS e	47.94	18.73
RS f	53.44	13.22
RS g	43.47	23.19
RS h	35.13	31.54

There seems to be a case for negotiation, especially if there are pre-specified maximum loan to value ratios, which in this case, avoid the worst case of immediate liquidation, and which might involve some generosity in LTV terms from debtholders in return for an equity stake (shared equity). But, given these parameter values, there is not much equity value to share, since equity is a way-out-of-the money call option, and the debtholders have now written a way-in-the-money put option. However, management and shareholders have some power against debtholders, since the worst strategy seems to be immediate liquidation. Very SSM might negotiate, in this case, along a very narrow range, for an alternative to shareholder determined $V_{B,}$ strategy i.

	A	B	C	D	E	F	G	H	I	J
1						**Real Debt Strategy i**				
2				Figure 10.10						
3		**BASE CASE**								
4		**ACCOUNTS**	**REAL 1**	**REAL 2**	**REAL 3**	**REAL 4**	**REAL 5**	**REAL 6**	**REAL 7**	
5	ASSETS	40.00	42.10	47.11	46.34	45.24	44.36	43.08	42.06	V*
6	DEBT	66.67	39.62	44.81	43.99	42.83	41.91	40.61	39.58	D*
7	EQ	-26.67	2.48	2.30	2.35	2.41	2.44	2.47	2.48	E*
8			WE CHOOSE	YOU CHOOSE	NEGOTIATE	NEGOTIATE	NEGOTIATE	NEGOTIATE	NEGOTIATE	
9	Retained Equity		**2.48**	**2.30**	**2.33**	**2.36**	**2.37**	**2.37**	**2.35**	
10	NOM V	40.00	40.00	40.00	40.00	40.00	40.00	40.00	40.00	
11	Coupon	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	
12	NOM Debt	66.67	66.67	66.67	66.67	66.67	66.67	66.67	66.67	
13	δ		0.00	0.00	0.00	0.00	0.00	0.00	0.00	
14	σ		0.20	0.20	0.20	0.20	0.20	0.20	0.20	
15	α		0.50	0.50	0.50	0.50	0.50	0.50	0.50	
16	τ		0.35	0.35	0.35	0.35	0.35	0.35	0.35	
17	r(f)		0.06	0.06	0.06	0.06	0.06	0.06	0.06	
18	OUTPUT									
19	V_B		32.50	30.03	30.44	31.01	31.45	32.05	32.52	
20	β_2		-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	-3.00	
21	LTV	**166.67%**	**158.35%**	**141.52%**	**143.86%**	**147.36%**	**150.30%**	**154.73%**	**158.52%**	
22			LTV MAX	2.22	2.19	2.15	2.12	2.08	2.05	
23			SHARED EQUITY	0	0.01	0.02	0.03	0.04	0.05	
24	DEBT LOSS + SH EQ	46.67	27.04	21.86	22.70	23.88	24.83	26.15	27.21	
25	CHANGE DEBT LOSS		**-19.62**	**-24.81**	**-23.97**	**-22.79**	**-21.84**	**-20.51**	**-19.46**	
26	LTV			$B$12/D5						
27	V_B			$B$6/D22						
28	DEBT LOSS + SH EQ		$B$6-C6	$B$6-D6+D23*D7						
29	CHANGE DEBT LOSS		**C24-$B$24**							

In Figure 10.10, column B shows debt loss in immediate bankruptcy. Column C shows debt loss if management chooses the bankruptcy trigger, which reduces the debt loss 19.62. Column D shows the debt loss if bondholders choose a trigger of 30.03, equal to a LTV nominal debt to real asset ratio (66.67/47.11)=141.5%. Assuming the SEC can continue to pay the coupon perpetually, and there are no other changes, suppose in a "fair" negotiation that neither shareholders nor debtholders are given complete power to determine V_B, but in return for small percentage of the equity given to the debtholders. Shareholders would be better off in REAL 5 (LTV=2.12), even where debtholders are given a 3% equity stake, since 97% share in the equity would be worth $2.37, and debtholders would be slightly better off than in the REAL 1 case, and lots better off than if liquidation were immediate. What is a fair solution?

CLASS EXERCISE: SIMPLE STRATEGY

Michael Flanagan wants to buy a modest house worth $100,000, taking out a sensible perpetual mortgage D from Arizona Safe Union (ASU) of $75,000 with a perpetual coupon of $3000, when the current riskfree rate is 4% (D=$3000/.04). He knows the house volatility is 20%, it would rent for $4000 per annum, but he agrees with the ASU that the real estate broker and legal fees in a foreclosure would be 50% of the house value. He cannot deduct mortgage interest from his personal taxable income, but the silly lender will give Michael the perpetual right to default, when he wants, as long as he keeps paying the mortgage interest. ASU will look at the current value of Michael's house only when granting the initial mortgage.

What is the current value of this mortgage to ASU? When should Michael hand over the keys to ASU? What is the value of this arrangement for Michael?

USEFUL FORMULAS

$$\beta_2 = \frac{1}{2} - \frac{(r-\delta)}{\sigma^2} - \sqrt{\left[\frac{(r-\delta)}{\sigma^2} - \frac{1}{2}\right]^2 + \frac{2r}{\sigma^2}} < 0$$

$$D^* = D + [(1-\alpha)V_B - D](\tfrac{V_B}{V})^{-\beta_2}$$

$$V^* = V - \alpha V_B(\tfrac{V_B}{V})^{-\beta_2}$$

$$E^* = V^* - D^*$$

$$V_B = D\left(\frac{-\beta_2}{1-\beta_2}\right)$$

EXERCISE 10.1

Goody Ghosh owns a house worth \$100,000, with a perpetual mortgage D of \$100,000 with a perpetual coupon of \$4000, when the current riskfree rate is 4% (D=\$4000/.04). He knows the house volatility is 20%, it would rent for \$4000 per annum, so β_2= -1. Foreclosure costs would be 50% of the house value. In a taxless world, what is the value of this arrangement for Goody?

EXERCISE 10.2

Goody Ghosh owns a house worth \$100,000, with a perpetual mortgage D of \$100,000 with a perpetual coupon of \$8000, when the current riskfree rate is 8% (D=\$8000/.08). He knows the house volatility is 20%, it would rent for \$6000 per annum, so β_2=-2. Foreclosure costs would be 50% of the house value. In a taxless world, what is the value of this arrangement for the lender?

EXERCISE 10.3

Cautious Flanagan wants to buy a classy house worth \$200,000, taking out a sensible perpetual mortgage D of \$100,000 with a perpetual coupon of \$4000, when the current riskfree rate is 4%. He knows the house volatility is 20%, it would rent for \$8000 per annum, so β_2=-1, foreclosure costs would be 50% of the house value. Taxes are nil. If house prices collapse, when should Cautious walk away?

PROBLEM 10.4

Goody Ghosh owns a house worth \$100,000, with a perpetual mortgage D of \$100,000 with a perpetual coupon of \$4000, when the current riskfree rate is 4% (D=\$4000/.04). He knows the house volatility is 30%, it would rent for \$4000 per annum, and foreclosure costs would be 40% of the house value. In a tax world where τ=.35, what is the value of this arrangement for Goody?

PROBLEM 10.5

Goody Ghosh owns a house worth \$100,000, with a perpetual mortgage D of \$133,333 with a perpetual coupon of \$8000, when the current riskfree rate is 6% (D=\$8000/.06). He knows the house volatility is 20%, it would rent for \$6000 per annum. Foreclosure costs would be 50% of the house value. In a tax world where τ=.35, what is the value of this arrangement for the lender if V_B is determined by the borrower? If V_B is at a MAX LTV_B of 3?

PROBLEM 10.6

Goody Ghosh owns a house worth $40,000, with a perpetual mortgage D of $66,666 with a perpetual coupon of $4000, when the current riskfree rate is 6% (D=$4000/.06). He knows the house volatility is 20%, it would rent for $400 per annum. Foreclosure costs would be 50% of the house value. In a tax world where τ=.30, what is the value of this arrangement for the lender if V_B is determined by the borrower? What is a “fair” arrangement for LTV_B in the range of 30 to 32.5 with equity sharing between the lender and the borrower?

REFERENCE

Leland. H.E. (1994), Corporate Debt Value, Bond Covenants and Optimal Capital Structure, *Journal of Finance*, 49(4): 1213-1252.

APPENDIX 10A DERIVATION OF D*, D'(V) and D''(V)

$$D(V\ \ ) = \frac{c}{r} + A_2 V^{\beta_2} \tag{10.3}$$

At $V = V_B$, $D = (1-\alpha)V_B$ (A1)

At $V = \infty$, $D = \frac{c}{r}$ (A2)

So

$$D(V_B) = \frac{c}{r} + A_2 {V_B}^{\beta_2} \quad = (1-\alpha)V_B \tag{A3}$$

$$A_{2=} \left[-\frac{c}{r} + (1-\alpha)V_B\right] V_B^{-\beta_2} \tag{A4}$$

$$D *= \frac{c}{r} + \left[(1-\alpha)V_B - \frac{c}{r}\right] (\frac{V_B}{V})^{-\beta_2} \qquad \text{(A5)=(10.6)}$$

$$D'(V) = \beta_2 \left[(1-\alpha)V_B - \frac{c}{r}\right] (\frac{V_B}{V})^{-\beta_2} V^{-1} \tag{A6}$$

$$D''(V) = (\beta_2^2 - \beta_2) \left[(1-\alpha)V_B - \frac{c}{r}\right] (\frac{V_B}{V})^{-\beta_2} V^{-2} \tag{A7}$$

APPENDIX 10B DERIVATION OF V* and E*

V*=V +TB -BC (B1)

BC=Bankruptcy Costs

$BC = A_2 V^{\beta_2}$

At $V = V_B$, $BC = \alpha V_B$ (B2)

At $V = \infty$, $BC = 0$ (B3)

So

$BC(V_B) = A_2 {V_B}^{\beta_2} \quad = \alpha V_B$ (B4)

$A_2 = [\alpha V_B] V_B^{-\beta_2}$ (B5)

$BC = [\alpha V_B](\frac{V_B}{V})^{-\beta_2}$ (B6)

TB=Tax Benefits

$TB = \frac{\tau c}{r} + A_2 V^{\beta_2}$

At $V = V_B$, $TB = 0 = \frac{\tau c}{r} + A_2 V^{\beta_2}$ (B7)

At $V = \infty$, $TB = \frac{\tau c}{r}$ (B8)

So

$$-A_2 {V_B}^{\beta_2} \quad = \frac{\tau c}{r} \tag{B9}$$

$$A_{2=} \frac{-\tau c}{r} V_B^{-\beta_2} \tag{B10}$$

$$TB = \frac{\tau c}{r} - \frac{\tau c}{r} (\frac{V_B}{V})^{-\beta_2} \tag{B11}$$

$$V *= V + \frac{\tau c}{r} \left[1 - (\frac{V_B}{V})^{-\beta_2} \right] - \alpha V_B (\frac{V_B}{V})^{-\beta_2} \qquad \text{(B12)=(10.7)}$$

E*=V*-D* (B13)

$$E *= V - \frac{(1-\tau)c}{r} + \left[(1-\tau)\frac{c}{r} - V_B \right] (\frac{V_V}{V})^{-\beta_2} \qquad \text{(B14)=(10.8)}$$